MA-101/1841

B.Tech (Semester-I) Exam.–2016

Mathematics-I

Time : Three Hours *Maximum Marks :* **100**

Note : Attempt questions from all sections.

SECTION - A

(Short-answer Type Questions)

Note : Attempt any ten questions. Each question carries 4 marks. $10\times4=40$

1. Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary.

2. Find the rank of the matrix

$$A=\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

3. Show that the set/matrix

$$S=\{(1,0,0),(1,1,0),(1,1,1),(0,1,0)\}$$

is linearly dependent.

4. Show that the matrix

$$A=\begin{bmatrix} -i & 3+2i & -2-i \\ -3+2i & 0 & 3-4i \\ 2-i & -3-4i & -2i \end{bmatrix}$$

is skew Hermitian matrix.

[P. T. O.

5. Trace the curve $x^3 + y^3 = 3axy$.

6. If $u = \tan^{-1}\left[\frac{x^3 + y^3}{(x-y)}\right]$ prove that $x\frac{du}{dx} + y\frac{du}{dy} = \sin 2u$.

7. Disuss the maximum and minimum of $x^2+y^2+6x+12$.

8. Find the expansion of the function $e^x\log(1+y)$ in a Taylor series in the neighbourhood of the point (0,0).

9. Find by triple integration, the volume of the paraboloid of revolution $x^2+y^2=4z$. Cut off by the plane $z = 4$.

10. Prove that $B(m.n) = B(m+1,n) + B(m,n+1)$.

11. Evaluate $\int_2^0 \int_0^{\sqrt{2x-x^2}} (x^2+y^2)dy.dx$

12. Evaluate $\iiint_R (x+y+z)\, dx\, dy\, dz$ where $R : 0 \leq x \leq 1,\ 1 \leq y \leq 2,\ 2 \leq z \leq 3$

13. Find the divergence of the vector field $\vec{V} = (x^2 - y^2)\hat{i} + 2xy\hat{j} + (y^2 - xy)\hat{k}$

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14. Use Green's Theorem to evaluate $\int_C (x^2+xy)dx+(x^2+y^2)\,dy$ Where C is the square formed by the lines $y = \pm 1$, $x = \pm 1$.

15. Given that $\vec{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$ at $t = 2$ and $\vec{r}(t) = 4\hat{i} - 2\hat{j} + 3\hat{k}$ at $t = 3$ find $\int_2^3 \vec{r} \cdot \frac{d\vec{r}}{dt}\,dt$.

SECTION - B

(Long Answer type questions)

Note : Attempt **any three** questions. Each question carries 20 marks. 20x3=60

1. Find the characteristic equation, verify Cayley Hamilton Theorem & hence find A^{-1} of the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$

2. A square matrix A is defined by $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ Find the modal matrix P and the resulting diagonal matrix D of A.

[P. T. O.

$\sqrt{d}$

3. (a) If $u(x+y) = x^2 + y^2$ then prove that $(dy/dx - du/dy)^2 = 4\,(1 - du/dx - du/dy)$

(b) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, find the Jacobian of x, y, z with respect to r, θ, ϕ.

4. State Stoke's Theorem & Verify this theorem for

$$F = (x^2+y^2)\hat{i} - 2xy\hat{j}$$

taken round the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$

5. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}}\, dz\,dy\,dx$

6. Change the order of integration and evaluate.

$$\int_0^\infty \int_x^\infty (e^{-y}/y)\,dy\,dx$$

$dy\,dx$

$\sqrt{2x-x^2}\,dx$

$\left[\frac{(2x-x^2)^{3/2}}{3}\right] dx$

$\left[\frac{(2x^2-x^4)^{3/2}}{3}\right]$

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